



TE TĀHUHU O TE MĀTAURANGA

Ministry of Education

Hei Tautoko i te Marautanga Pāngarau

# Te Tūponotanga



# He Kupu Whakataki

Ko te kaupapa ia o tēnei pukapuka, ko te āta whakaatu i ngā **ariā matua** o tēnei wāhanga o te pāngarau, te Tūponotanga, me ngā whanaketanga ako, hei ekenga taumata mā ā tātou ākongā. Āpiti atu ki tēnā, ko te whakaatu i ētahi ngohe e hāngai ana ki tēnā taumata ki tēnā taumata o te marautanga, me ngā hononga ki te **Mahere Tau**<sup>1</sup> me ngā Whanaketanga Pāngarau<sup>2</sup>.

main ideas

Number Framework

He mea tuhi tēnei pukapuka i te tau 2010 hei āwhina i te whakaūnga o Ngā Whanaketanga Pāngarau i ō tātou kura reo Māori. Ko te whakaaro kia noho tauira tonu tēnei pukapuka mō tētahi wā, waihoki ka tāpiri haerehia ētahi atu ngohe, ētahi atu whakamārama ka puta i ngā kura me ngā kaitakawaenga i ngā tau e rua e tū mai ana.

Ko tā tēnei whenu, he tiroiro i ngā **putanga** o tētahi mahi, o tētahi āhuetanga rānei (ka kīia tēnei mahi, tēnei āhuetanga rānei, he **pāpono**). He mea nui te tiroiro me te tātai i te **tūponotanga** o ngā putanga o tētahi pāpono i roto i te ao **pakihi**, i te ao hākinakina, i roto hoki i ētahi kēmu maha, pērā i te Monopoly me ētahi atu. Ka whakamahia he hautau kei waenganui i te kore me te tahi, he ōrau rānei mai i te 0% ki te 100% hei whakaatu i te tūponotanga o tētahi putanga.

outcome

event

probability/chance

business

I te tīmatanga o te ako i te tūponotanga ka tino whai wāhi mai ngā momo kīanga pēnei i te ‘kāore e kore’, te ‘kore rawa’, me te ‘ākene pea’. E tino hono ana te tūponotanga ki ētahi atu whenu o te pāngarau. Hei tauira:

## Te Tau

Ka whakamahia he hautau hei whakaatu i te **tūponotanga** o tētahi **putanga**:

probability, outcome,

toss, coin

roll

dice

- 50% te tūponotanga ka puta he whiore ina **piua** he **uka**
- $\frac{1}{3}$  te tūponotanga ka puta he rima, he ono rānei ina **pīroria** he **mataono tau**

Ka tatauria te maha o tētahi putanga i tētahi **whakamātau tūponotanga**:

probability experiment

Ngā Pīroritanga Mataono Tau

| putanga | maha |
|---------|------|
| 1       |      |
| 2       |      |
| 3       |      |
| 4       |      |
| 5       |      |
| 6       |      |

<sup>1</sup> Tirohia: *Te Poutama Tau, Pukapuka Tuatahi*. Te Whanganui-a-Tara: Te Tāhuhu o te Mātauranga. 2007.

<sup>2</sup> Tirohia: *Whanaketanga Pāngarau: He aratohu mā te pouako*. Te Whanganui-a-Tara: Te Tāhuhu o te Mātauranga. 2010.

## Te Tauira me te Pānga

Mā te **whārite taurangi** e tātaihia ai te tūponotanga o tētahi putanga:

algebraic equation

$$T = \frac{P}{k}$$

T = tūponotanga  
P = te maha o ngā putanga  
k = te katoa o ngā putanga e taea ana

Mēnā e pīrorihia ana tētahi mataono tau, ka whakamahia te whārite nei hei tātai i te tūponotanga ka puta he **taukehe**. Ko te 1, te 3 me te 5 ngā putanga, arā e 3 ērā. E ono katoa ngā putanga e taea ana (1, 2, 3, 4, 5, 6):

odd number

$$\begin{aligned} T &= \frac{P}{k} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

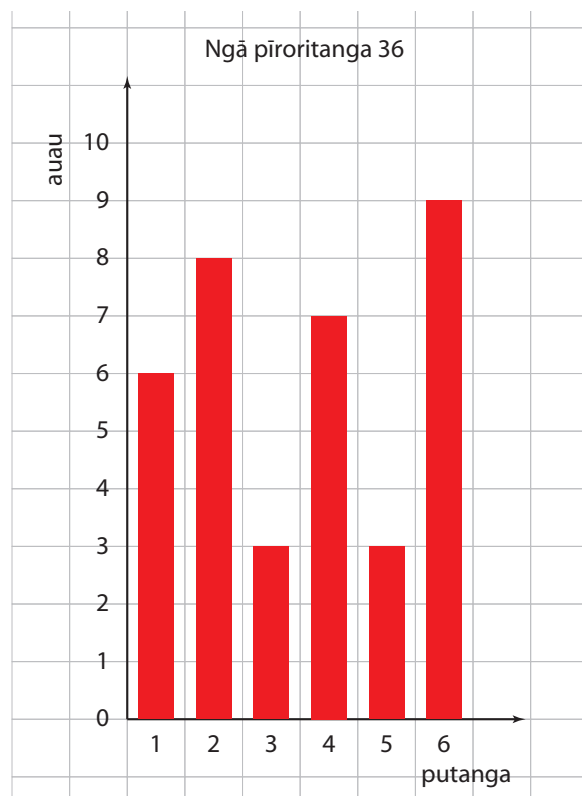
## Te Tauanga

Ko tētahi wāhanga matua o te **whakamātau tūponotanga**, he kohikohi **raraunga**, he whakaatu anō i aua raraunga ki tētahi kauwhata. Hei tauira, ko te pīrori mataono tau:

probability experiment data

E 36 ngā pīroritanga mataono tau

| putanga | tatau | auau |
|---------|-------|------|
| 1       | I     | 6    |
| 2       |       | 8    |
| 3       |       | 3    |
| 4       |       | 7    |
| 5       |       | 3    |
| 6       |       | 9    |



## **Hei Whakamihi**

Nā He Kupenga Hao i te Reo ngā tuhinga o tēnei pukapuka. E whakamihi ana ki ngā puna kōrero e whai ake nei, inā te ohonga ake o te hinengaro.

Te Reo Pāngarau: Putanga Tuarua. (2010). Papaioea: He Kupenga Hao i te Reo.

Van de Walle, J.A. (2007). Elementary and Middle School Mathematics: Teaching Developmentally. Boston: Pearson.

Ngā rauemi katoa o Te Poutama Tau

Ngā rauemi o <http://www.nzmaths.co.nz/>

Ngā pukapuka tauanga o te kāhui pukapuka He Tau Anō Te Tau, nā Te Tāhuhu o te Mātauranga i whakaputa.

# Te Tūponotanga: Ngā Ariā Matua

1. E toru ngā takenga mai o te tūponotanga o tētahi **putanga**:

outcome

- Ko te titiro whakamuri ki taua putanga i ngā wā o mua. Hei tauira, ko te tūponotanga ka eke ki tua atu i te 90% te ōrau o ngā ūhanga a Irene i tana kēmu poitarawhiti. Ina titiro whakamua ki ana kēmu tekau kua pahure tata nei, ka tino kitea i te nuinga o aua kēmu ka eke ki tua atu i te 90%, nō reira he nui te tūponotanga ka pērā anō ia i te kēmu e heke mai ana.

| kēmu | 1  | 2  | 3   | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|------|----|----|-----|----|----|----|----|----|----|----|
| ōrau | 92 | 94 | 100 | 98 | 93 | 88 | 90 | 95 | 97 | 94 |

Ko te mōhio o te tangata ki te horopaki o te putanga te mea matua hei **tautohu** i te tūponotanga o taua putanga. Pēnā he tangata mahi māra kua roa e noho ana ki tētahi wāhi, e tino mōhio ana ki te tūponotanga ka tau te hukapapa i muri i te marama o Mahuru, kāore rānei.

identify

- Ko te whakahaere **whakamātau tūponotanga**. Hei tauira, ina piua tētahi **pine panapana**, he aha te tūponotanga ka tau ki tōna upoko, ka tītaha rānei te tau:

probability experiment  
drawing pin



Ka whakahaerehia te whakamātau hei kimi i ngā tūponotanga nei. Arā, ka 1000 pea ngā piunga o tētahi pine panapana, me te tuhi anō i te maha o ia **putanga** (upoko, tītaha rānei). Nā runga i ēnei **raraunga** ka whakatauria ngā tūponotanga. Hei tauira:

outcome, data

| te putanga | upoko | tītaha |
|------------|-------|--------|
| te maha    | 700   | 300    |

$$T(\text{upoko}) = \frac{700}{1000} = \frac{7}{10} = 70\%$$

$$T(\text{tītaha}) = \frac{300}{1000} = \frac{3}{10} = 30\%$$

- Ko te tātai i te tūponotanga i runga anō i te **whakaaro arorau**. Hei tauira, ko te whakaaro arorau e mea ana ina piua he uka, he ōrite te tūponotanga ka puta he whiore, ka puta rānei he upoko. Arā, kotahi o roto i te rua ( $\frac{1}{2}$ ) te tūponotanga o tētahi upoko, he pērā anō te tūponotanga o tētahi whiore:

logical reasoning

$$T(u) = \frac{1}{2}$$

$$T(wh) = \frac{1}{2}$$

Mēnā e rua ngā uka ka piua, e toru ngā putanga e taea ana – ko ētahi upoko e rua ( $u, u$ ), ko ētahi whiore e rua ( $wh, wh$ ), ko tētahi upoko me tētahi whiore rānei ( $u, wh$ ):

|       |    |       |       |
|-------|----|-------|-------|
|       |    | uka 1 |       |
|       |    | u     | wh    |
|       |    |       |       |
| uka 2 | u  | u,u   | u,wh  |
|       | wh | u,wh  | wh,wh |

Ko te whakaaro arorau e mea ana he ōrite te tūponotanga o ia putanga e whakaaturia ana ki te tūtohi, nō reira:

$$T(e \text{ rua ngā upoko}) = \frac{1}{4}$$

$$T(e \text{ rua ngā whiore}) = \frac{1}{4}$$

$$T(kotahi te upoko, kotahi te whiore) = \frac{2}{4} = \frac{1}{2}$$

2. I te nuinga o te wā, ka rerekē ngā putanga o tētahi whakamātau i ngā putanga ka tātaihia i runga i te whakaaro arorau. Ka kīia tēnei ko te **taurangirangi** o tēnei mea te tūponotanga. Hei tauira, ko te tātaianga e mea ana, mēnā e 20 ngā piunga uka, ka 10 ngā upoko, ka 10 anō ngā whiore. Engari ka taurangirangi ngā putanga o te whakamātau:

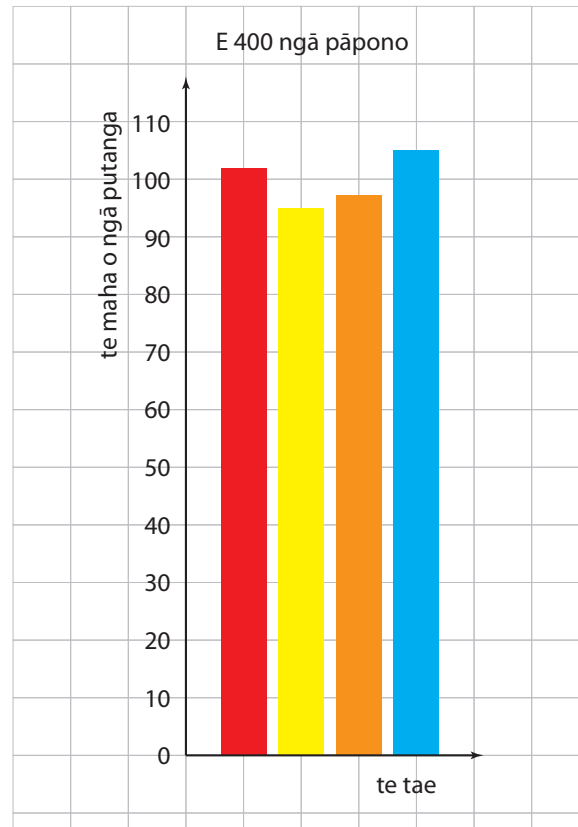
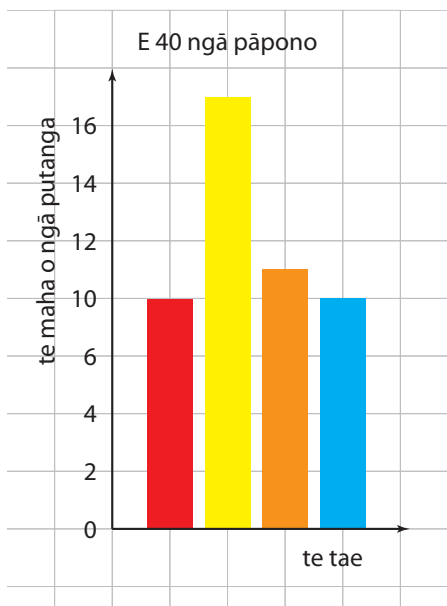
variability

| whakamātau | 1  | 2  | 3  | 4  |
|------------|----|----|----|----|
| upoko      | 11 | 8  | 10 | 7  |
| whiore     | 9  | 12 | 10 | 13 |

3. Ina maha haere ngā **pāpono** o tētahi whakamātau, kāore e pērā rawa te **taurangirangi**. Hei tauira, mēnā e whā ngā poi paku kei roto i tētahi pēke, he rerekē ngā tae, ā, ka tangohia **matapōkerea** tētahi, koinei ngā putanga tātai me ētahi tauira putanga whakamātau:

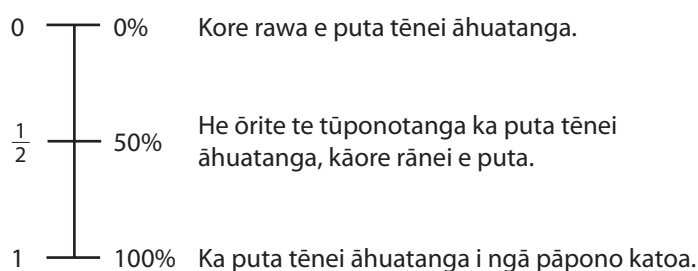
events  
variability  
random

|                  | whero      |       | kōwhai     |       | karaka     |       | kahurangi  |       |
|------------------|------------|-------|------------|-------|------------|-------|------------|-------|
|                  | whakamātau | tātai | whakamātau | tātai | whakamātau | tātai | whakamātau | tātai |
| e 40 ngā pāpono  | 8          | 10    | 15         | 10    | 9          | 10    | 8          | 10    |
| e 400 ngā pāpono | 103        | 100   | 95         | 100   | 98         | 100   | 104        | 100   |



- Kāore te putanga o tētahi **pāpono** e pā ki te tūponotanga o tētahi putanga i muri mai. Hei tauira, mēnā e ono ngā piunga uka, ā, e ono ngā putanga o te upoko, ko te 50% tonu te tūponotanga ka puta anō he upoko i te piunga tuawhitu.
- Ka whakamahia te hautau kei waenganui i te 0 me te 1, te ōrau rānei kei waenganui i te 0% me te 100% hei tautohu tūponotanga:

event



- Ko te **tūwehe** tētahi atu momo whakaatu i te tūponotanga. Arā, ko te whakamahi i te **ōwehenga** hei tautohu i te tūponotanga. Hei tauira, ina pīroria tētahi mataono tau, ko te 1:5 te tūwehe ka puta te ono. Kotahi te ono o roto i ngā **putanga** katoa e taea ana, ā, e rima ērā atu o ngā putanga.

odds  
ratio

outcome

7. Mēnā he ōrite te tūponotanga o ia putanga i tētahi pāpono, ka kīia he **pāpono tōkeke** tērā. Hei tauira, ko te tae o tētahi poi ka tangohia matapōkerea i tētahi pēke, mēnā kotahi te poi o ia tae.

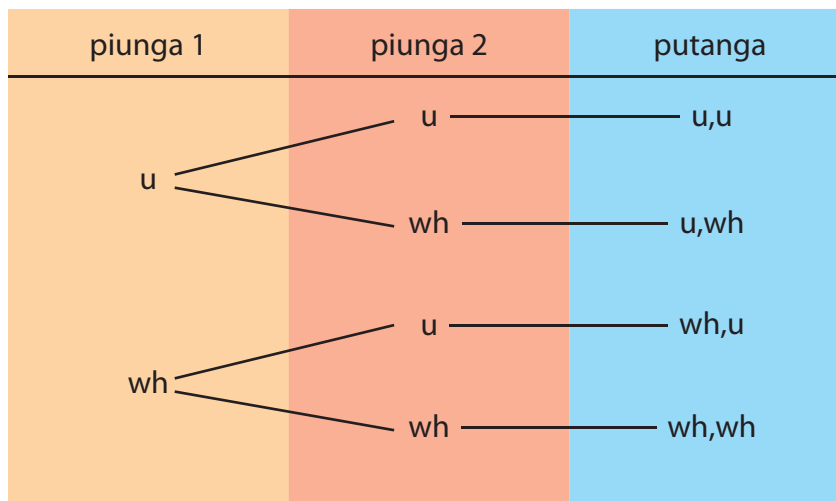
fair event

8. Mēnā kāore e ōrite ana te tūponotanga o ia putanga i tētahi pāpono, ka kīia he **pāpono haukume** tērā. Hei tauira, ko te tae o tētahi poi ka tangohia matapōkerea i tētahi pēke mēnā e rua ngā poi whero, ā, kotahi te poi o ērā atu o ngā tae.

biased event

9. Ka whakaaturia ngā putanga o tētahi **pāpono raupapa** (arā ētahi pāpono e rua e whai mai ana tētahi i tētahi) ki te **hoahoa rākau**. Hei tauira, ko te piu i ētahi uka e rua:

combined event  
tree diagram



10. Ka kīia he **wehe kē** ētahi pāpono, he **whakawhirinaki** ētahi. Hei tauira, ko te tango matapōkere i ētahi poi e rua i tētahi pēke. Mēnā ka whakahokia te poi tuatahi, he pāpono wehe kē, nā te mea kāore te putanga o tētahi tangohanga e pā ki te putanga o tētahi. Engari ki te kore e whakahokia te poi tuatahi, ka pā tērā ki te tangohanga tuarua, nā te mea kua ruarua ake ngā poi mō te tangohanga tuarua. Ka kīia he pāpono whakawhirinaki.

independent,  
dependent



## Te Tūponotanga: He Tauira Mahere

| Taumata 1       | Te Mātauranga Tūponotanga   | Te Whakaoti Rapanga Tūponotanga  | Te Reo Matatini o te Tūponotanga                                      |
|-----------------|---|--|---|
| Whanaketanga 1a | Karekau he whanaketanga e hāngai ana ki te tūponotanga i tēnei taumata        |  |   |
| Whanaketanga 1c | E mōhio ana ki ngā putanga katoa e taea ana i tētahi pūāhua tūponotanga māmā. | <p>I tētahi pūāhua tūponotanga māmā, ka tautohua te putanga he nui te tūponotanga, te putanga he iti te tūponotanga, mēnā rānei he āhua ōrite te tūponotanga.</p> <p>Ka whakahaere whakamātau tūponotanga māmā me te tuhi anō i ngā putanga ki tētahi tūtohi.</p> <p>Ka matapae i te putanga o tētahi āhuatanga i runga anō i te mōhio ki te putanga o taua āhuatanga i mua. Hei tauira, ko te whiti mai o te Rā āpōpō mēnā kua toru rangi e ua ana.</p> | E mōhio ana ki ngā kupu tūponotanga nei: tūponotanga, putanga, tūtohi |

| Taumata 2          | Te Mātauranga<br>Tūponotanga  | Te Whakaoti Rapanga<br>Tūponotanga   | Te Reo Matatini o<br>te Tūponotanga   |
|--------------------|---|--|---|
| Whanaketanga<br>2a | <p>E mōhio ana ki te matapōkere o ngā putanga i tētahi pūāhua tūponotanga.</p> <p>E mōhio ana ki te wehe kētanga o ngā pāpono. Arā, kāore te putanga o tētahi pāpono e pā ki te putanga o te pāpono e whai mai ana. Hei tauira, ina puta he upoko i tētahi piunga uka, he ōrite tonu te tūponotanga ka puta he upoko, he whiore ranei i te piunga o muri mai.</p> | <p>Ka whakahaere whakamātau tūponotanga māmā, me te tuhi anō i ngā putanga ki tētahi tūtohi.</p> <p>Ka matapae i ngā putanga o tētahi whakamātau tūponotanga i runga anō i te whakaaro arorau.</p> | <p>E mōhio ana ki te reo māmā o ia rā hei whakaahua tūponotanga, pēnei i ēnei:</p> <ul style="list-style-type: none"> <li>• kāore e kore</li> <li>• ākene pea</li> <li>• kore rawa</li> <li>• ...</li> </ul> <p>E mōhio ana ki ngā kupu tūponotanga nei: whakamātau, matapae</p>                              |
| Whanaketanga<br>2e | <p>E mōhio ana ki te tikanga o te āwhata hautau (mai i te 0 ki te 1) hei whakaatu tūponotanga. Arā, ko te 0 e mea ana e kore rawa e puta tētahi āhuatanga, ko te tahi e mea ana, āe marika ka puta taua āhuatanga. Ko te <math>\frac{1}{2}</math> e mea ana he ōrite te tūponotanga ka puta taua āhuatanga, kāore rānei e puta.</p>                               | <p>Ka whakahaere whakamātau tūponotanga māmā, me te whakaatu anō i ngā putanga ki tētahi whakaari raraunga.</p> <p>Ka whakamāori i te whakaari raraunga o tētahi whakamātau tūponotanga.</p>       | <p>E mōhio ana ki te whakaatu i ngā putanga o tētahi whakamātau tūponotanga ki tētahi whakaari raraunga, pērā i te:</p> <ul style="list-style-type: none"> <li>• kauwhata pou</li> <li>• kauwhata ira</li> <li>• kauwhata whakaahua</li> </ul> <p>E mōhio ana ki ngā kupu tūponotanga nei: pāpono, āwhata</p> |

| Taumata 3       | Te Mātauranga Tūponotanga  | Te Whakaoti Rapanga Tūponotanga  | Te Reo Matatini o te Tūponotanga   |
|-----------------|--|--|--|
| Whanaketanga 3a | <p>E mōhio ana ki te tūkanga o tētahi hautau māmā hei whakaatu i te tūponotanga o tētahi putanga. Hei tauira:</p> <p><math>T = \frac{1}{2}</math>: He ōrite te tūponotanga ka puta tēnei āhuatanga, kāore rānei e puta.</p> <p><math>T = \frac{3}{4}</math>: He nui te tūponotanga ka puta tēnei āhuatanga</p> <p><math>T = \frac{1}{4}</math>: He iti te tūponotanga ka puta tēnei āhuatanga.</p> <p>E mōhio ana ki te whiriwhiri i te tūponotanga o ngā putanga i tētahi pūāhua tūponotanga māmā.</p> <p>E mōhio ana ki te taurangirangi o ngā putanga o tētahi whakamātau tūpono-tanga, kāore e whai tōtika i ngā tātaitunga tūponotanga.</p> | <p>Ka whakahaere whakamātau tūponotanga, me te whakaatu anō i ngā putanga.</p> <p>Ka matapae i ngā putanga o tētahi whakamātau tūponotanga i runga i te whakaaro arorau.</p> <p>Ka tātai i te tūponotanga o ngā putanga o tētahi whakamātau tūponotanga.</p> | E mōhio ana ki ngā kupu tūponotanga nei: matapōkere  |
| Whanaketanga 3e | <p>E mōhio ana mēnā he wehe kē, he whakawhirinaki rānei ngā putanga o tētahi pāpono raupapa.</p> <p>E mōhio ana ki te whakaatu tūponotanga ki tētahi hoahoa rākau.</p>   | <p>Ka tātai i te tūponotanga o ngā putanga o tētahi pāpono raupapa, ka whakahaere whakamātau tūponotanga, me te whakamārama anō i te taurangirangi o ngā putanga matapae me ngā putanga whakamātau.</p>  | <p>E mōhio ana ki te whakaatu pāpono raupapa ki te hoahoa rākau.</p> <p>E mōhio ana ki ngā kupu tūponotanga nei: hoahoa rākau, pāpono raupapa, pāpono wehe kē, pāpono whakawhirinaki</p> |

| Taumata 4       | Te Mātauranga Tūponotanga   | Te Whakaoti Rapanga Tūponotanga   | Te Reo Matatini o te Tūponotanga   |
|-----------------|---|---|--|
| Whanaketanga 4a | E mōhio ana ki te whakaatu tūponotanga hei ōrau.  | <p>Ka whakaputa whakaaro ki te tōkeke, te haukume rānei o tētahi pūāhua tūponotanga, i runga anō i ngā tātaitanga me te whakaaro arorau.</p> <p>Ka whakahaere whakamātau tūponotanga:</p> <ul style="list-style-type: none"> <li>• ka matapae i ngā putanga</li> <li>• ka tātai i ngā tūponotanga</li> <li>• ka whakaatu i ngā putanga ki te whakaari raraunga</li> <li>• ka whakamāori i ngā hua o te whakamātau.</li> </ul> | E mōhio ana ki ngā kupu tūponotanga nei: tōkeke, haukume   |
| Whanaketanga 4e | <p>E mōhio ana ki ngā take e taurangirangi ai ngā putanga o tētahi whakamātau tūponotanga.</p> <p>E mōhio ana ki te iti haere o te taurangirangi i te nui haere o ngā pāpono o te whakamātau.</p> | <p>Ka whakahaere whakamātau tūponotanga:</p> <ul style="list-style-type: none"> <li>• ka matapae i ngā putanga</li> <li>• ka tātai i ngā tūponotanga</li> <li>• ka whakaatu i ngā putanga ki te whakaari raraunga</li> <li>• ka whakatairite i te tuari o ngā putanga matapae me ngā putanga whakamātau.</li> <li>• ka whakamārama i te taurangirangi o ngā putanga whakamātau</li> </ul>                                     | <p>E mōhio ana ki te whakaatu i te tuari o ngā putanga o tētahi whakamātau tūponotanga ki ngā momo whakaari raraunga e hāngai ana</p> <p>E mōhio ana ki ngā kupu tūponotanga nei: tuari, taurangirangi</p> |

| Taumata 5          | Te Mātauranga<br>Tūponotanga   | Te Whakaoti Rapanga<br>Tūponotanga  | Te Reo Matatini o te<br>Tūponotanga   |
|--------------------|--|---|---|
| Whanaketanga<br>5a | E mōhio ana ki te tātai<br>i te tūponotanga o ngā<br>putanga i tētahi pūāhua<br>tūponotanga papa-maha. | Ka hangaia he pūāhua whakatau<br>hei tūhura i tētahi pūāhua<br>tūturu.<br><br>Ka whakahaere whakamātau<br>tūponotanga, ka whakaputa<br>whakaaro ki ōna wāhanga katoa. | E mōhio ana ki ngā kupu<br>tūponotanga nei:<br>pūāhua whakatau, pūāhua<br>tūponotanga papa-maha |